Math 116 Section 04

Quiz 5 Name _____
August 04, 2005
Instructor: Charles Cuell Student Number _____

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions. ***There are two questions***.

(1) Find the area between the curves $x - y = 7$ and $x = 2y^2 - y + 3$

- Solve the first equation for x to get $x = y + 7$. Set the two equations equal to find the intersection points:

$$\begin{aligned} y + 7 &= 2y^2 - y + 3 \\ y^2 - y - 2 &= 0 \\ (y-2)(y+1) &= 0 \end{aligned}$$

  So $y = -1, 2$.
- Find which curve is higher by using the test point $y = 0$. For $x = y + 7$, when $y = 0$, $x = 7$. For $x = 2y^2 - y + 3$, $x = 3$.
- The area is then

$$\begin{aligned} A &= \int_{-1}^{2} (7 + y - (2y^2 - y + 3))\, dy \\ &= \int_{-1}^{2} (-2y^2 + 2y + 4)\, dy \\ &= \left(-\frac{2}{3}y^3 + y^2 + 4y \right) \Big|_{-1}^{2} \\ &= y \left(-\frac{2}{3}y^2 + y + 4 \right) \Big|_{-1}^{2} \\ &= 2\left(-\frac{2}{3}2^2 + 2 + 4 \right) - (-1)\left(-\frac{2}{3}(-1)^2 + (-1) + 4 \right) \\ &= 9 \end{aligned}$$

(2) A solid object is 3m high. The area of a cross section x metres above the base is $2x$ square metres. Find the volume of the solid.

The general formula is $V = \int_{x_0}^{x_1} A(x)\,dx$. In this case, $x_0 = 0$, $x_1 = 3$ and $A(x) = 2x$. So

$$\begin{aligned} V &= \int_0^3 2x\,dx \\ &= x^2\Big|_0^3 \\ &= 9 \end{aligned}$$